Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 21, 2004*
Accepted: *December 16, 2004*

Supersymmetric strings and waves in $D = 3$, $N = 2$ matter coupled gauged supergravities

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Abstract: We construct new 1/2 supersymmetric solutions in $D = 3$, $N = 2$, matter coupled, U(1) gauged supergravities and study some of their properties. In the most general case they represent a string superposed with gravitational and Chern-Simons electromagnetic waves. The waves are attached to the string and the solution satisfies an electromagnetic self-duality relation. When the sigma model is non-compact it interpolates between an asymptotically Kaigorodov space and a naked singularity. For the compact sigma model there is a regular horizon with the Kaigorodov geometry and asymptotically it is either Minkowskian or a pp-wave. When the sigma manifold is flat our solutions describe either AdS_3 or Kaigorodov space or a pp-wave in AdS_3.

Keywords: Supergravity Models, AdS-CFT and dS-CFT Correspondence, Penrose limit and pp-wave background.

Contents

1. Introduction

Supersymmetric solutions of supergravity theories have played a major role in many of the recent advances in string/M theory. Among supergravity theories interest in the gauged ones increased dramatically after the advent of the AdS/CFT duality [1, 2, 3] since generically they possess scalar fields with potentials which have some AdS extrema. To test this conjecture, gravity theories in three dimensional anti-de Sitter space (for a review see [4]) is especially appropriate because they are claimed to be related to two dimensional CFT's and such conformal field theories are the best understood ones.

It is clearly desirable to go beyond the supergravity approximation in AdS/CFT correspondence and recently it is understood that pp-wave backgrounds provide such an opportunity [5]. Plane waves (which are a subset of pp-waves) can arise by taking the Penrose-Güven limit [6, 7] of $AdS_p \times S^q$ backgrounds [8] and string theory in many of them turns out to be exactly solvable [9] (for an up-to-date review and more references see [10]). A way to improve our understanding of such spacetimes is to construct asymptotically pp-wave solutions. Such black strings were recently studied in [11, 12, 13, 14].

Motivated by these, in this paper we find new superysmmetric solutions in the matter coupled $D = 3$, $N = 2$, U(1) gauged supergravities and study some of their properties. This model was constructed in [15] and admits both compact and non-compact sigma model manifolds. There is also a well-defined flat sigma model limit. The theory contains only a Chern-Simons gauge field and no Maxwell term. The only known supersymmetric solutions of this model are static, uncharged strings [15] and vortices [16]. Among our solutions we have stationary and charged generalizations of the strings given in [15]. More-

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over, by carefully analyzing Killing spinor and field equations we show that it is possible to add two types of waves to these strings. Some time ago Garfinkle and Vachaspati developed a technique [17, 18] which allows one to introduce waves to an already constructed solution with a null Killing vector. Our first type of wave turns out to be exactly the one obtained by this method starting from the string solution. The second type exists only when there is a non-trivial radial dependence in the Chern-Simons vector field and therefore can be regarded as an electromagnetic wave. It is not possible to separate the waves from the string and the solution satisfies an electromagnetic self-duality relation. The charge of the solution can be set to zero by choosing the scalar field and the Killing spinor real.

The scalar fields are not affected with these additional waves but the global properties of the metric are quite sensitive to them. For example when the sigma model manifold is non-compact, asymptotic geometry for the solution without waves approaches to AdS_3 whereas with waves it becomes the Kaigorodov space [19]. This is a homogeneous Einstein space which describes a pp-wave in AdS and in three dimensions it is equivalent to the extremal BTZ black hole [20]. This space has already appeared in the AdS/CFT context [21, 22] where a duality between a supergravity theory in the Kaigorodov space and a CFT living on its boundary was proposed (see also [23, 24, 25, 26]). This boundary is related to the usual AdS boundary by an infinite Lorentz boost [21]. For the compact sigma model our solutions exhibit an event horizon whose geometry is deformed from AdS to Kaigorodov space by the presence of the waves as was observed for $M2, M5$ and $D3$ branes in [21]. All curvature invariants are finite at the horizon and we show that geodesics never cross it. When the electromagnetic wave is absent the asymptotic geometry is Minkowskian but it is a pp-wave otherwise. For the flat sigma model our solutions describe either AdS_3 or Kaigorodov space or a pp-wave in AdS_3.

The plan of this paper is as follows. In section 2 we begin with a review of the $N = 2$ gauged supergravity with matter. In section 3 we derive our supersymmetric string and wave solutions. We conclude in section 4 with some comments and future directions. A brief introduction to the Garfinkle-Vachaspati solution generating method together with its application to our string solution is given in the appendix.

2. The model

In this paper we consider $N = 2$, U(1) gauged supergravity in $D = 3$ interacting with an arbitrary number of matter multiplets which was constructed in [15] using Noether's procedure. Its higher dimensional origin is yet to be discovered. The boundary symmetries of this theory were studied in [28] and its extension by including a Fayet-Iliopoulos term was given in [16]. Holographic RG flows in this model were analyzed in [29]. There is another $N = 2$ theory constructed in [30] where scalars are not charged with respect to the U(1) R-symmetry group. Therefore in [30] there is only a cosmological constant and no scalar potential. The connection between these two models for the flat sigma model was described in [28]. Let us also mention that the model we consider in this paper [15] is a member of a class of theories called abelian Chern-Simons Higgs models coupled to gravity (see [16, 31] and references therein). The field content of the theory is:

- The supergravity multiplet: $\{e_\mu{}^a,\ \psi_\mu,\ A_\mu\}$
- The scalar multiplet (K copies): $\{\phi^\alpha,\ \lambda^r\}$

All fields except the graviton $e_\mu{}^a$ and the gauge field A_μ are complex. In [15], the following sigma model manifolds M were considered:

$$M_+ = CP^K = \frac{\mathrm{SU}(K+1)}{\mathrm{SU}(K)\times\mathrm{U}(1)}\,, \qquad M_- = CH^K = \frac{\mathrm{SU}(K,1)}{\mathrm{SU}(K)\times\mathrm{U}(1)}\,. \tag{2.1}$$

Note that U(1) is the R-symmetry group. We define the parameter $\epsilon = \pm 1$ to indicate the manifolds $M_\pm$. In this paper we choose $K = 1$ and consider the following cases, $S^2 = \mathrm{SU}(2)/\,\mathrm{U}(1)$ and $H^2 = \mathrm{SU}(1,1)/\,\mathrm{U}(1)$. The bosonic part of the lagrangian is[1]

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{4}R - \frac{1}{16ma^4}\frac{\epsilon^{\mu\nu\rho}}{\sqrt{-g}}A_\mu\partial_\nu A_\rho - \frac{|D_\mu\phi|^2}{a^2(1+\epsilon|\phi|^2)^2} - V(\phi)\right), \tag{2.2}$$

where $D_\mu\phi = (\partial_\mu - i\epsilon A_\mu)\phi$ and the potential is given by

$$V(\phi) = 4m^2a^2C^2\left(|S|^2 - \frac{1}{2a^2}C^2\right). \tag{2.3}$$

Functions C and S are defined as

$$C = \frac{1-\epsilon|\phi|^2}{1+\epsilon|\phi|^2}\,, \qquad S = \frac{2\phi}{1+\epsilon|\phi|^2}\,. \tag{2.4}$$

Note that the following algebraic relations hold:

$$|\phi|^2 = \frac{\epsilon(1-C)}{(1+C)}\,, \qquad \epsilon|S|^2 = 1 - C^2\,. \tag{2.5}$$

The constant "a" is the characteristic curvature of $M_\pm$ (e.g. $2a$ is the inverse radius in the case of $M_+ = S^2$). The gravitational coupling constant κ has been set equal to one and $-2m^2$ is the AdS_3 cosmological constant. Unlike in a typical AdS supergravity coupled to matter, the constants κ, a, m are not related to each other for non-compact scalar manifolds, while a^2 is quantized in terms of κ in the compact case so that κ^2/a^2 is an integer [15]. When $\epsilon = -1$ for all a^2 there is a supersymmetric AdS vacuum at $\phi = 0$ and a non-supersymmetric but stable (it satisfies Breitenlohner-Freedman bound [32]) AdS vacuum for $1/2 < a^2 < 1$. When $\epsilon = 1$ there are supersymmetric AdS, Minkowski and non-supersymmetric de Sitter vacua (see figure 1).

[1] Our conventions are as follows: We take $\eta_{ab} = (-,+,+)$ and $\epsilon^{\mu\nu\rho} = \sqrt{-g}\gamma^{\mu\nu\rho}$. In coordinate basis a convenient representation for γ^a matrices is $\gamma_0 = i\sigma^3, \gamma_1 = \sigma^1, \gamma_2 = \sigma^2$ with $\epsilon^{012} = 1$. Here 0,1,2 refer to the tangent time, radial and theta directions, respectively, and γ^2 is the charge conjugation matrix.

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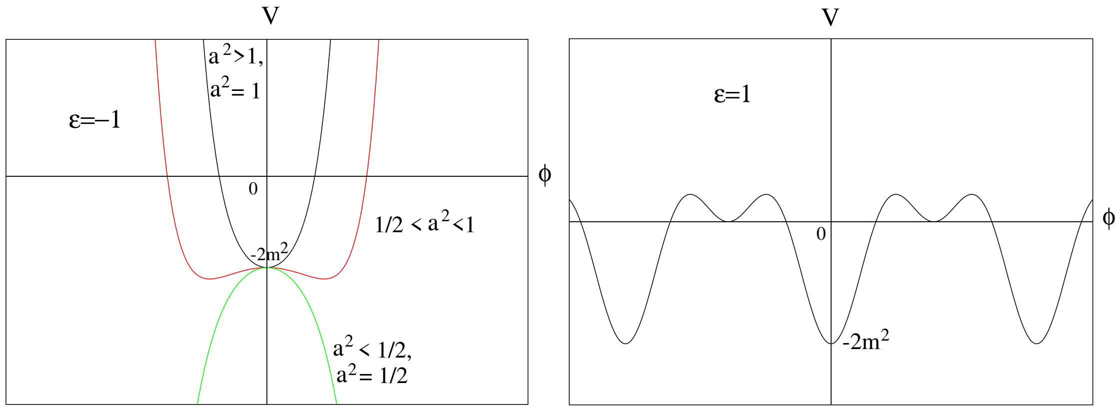

Figure 1: The scalar potential V plotted with respect to ϕ.

The nonlinear scalar covariant derivative P_μ and the U(1) connection Q_μ are defined as

$$P_\mu = \frac{2\partial_\mu\phi}{1+\epsilon|\phi|^2} - i\epsilon A_\mu S\,,$$

$$Q_\mu = \frac{i\phi \overset{\leftrightarrow}{\partial_\mu} \phi^*}{1+\epsilon|\phi|^2} + A_\mu C\,. \tag{2.6}$$

The bosonic field equations that follow from the lagrangian (2.2) are

$$R_{\mu\nu} = \frac{1}{a^2}P_{(\mu}P^*_{\nu)} + 4V g_{\mu\nu}\,, \tag{2.7}$$

$$\epsilon^{\mu\nu\rho}F_{\nu\rho} = -4\epsilon i m a^2\sqrt{-g}\,[P^\mu S^* - (P^\mu)^* S]\,, \tag{2.8}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}P_\nu\right) = i\epsilon Q_\mu P^\mu + 2a^2\left(1+\epsilon|\phi|^2\right)\frac{\partial V}{\partial\phi^*}\,. \tag{2.9}$$

The supersymmetric version of the lagrangian (2.2) is invariant under the following fermionic supersymmetry transformations

$$\delta\psi_\mu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - \frac{i}{2a^2}Q_\mu\right)\varepsilon + m\gamma_\mu C^2\varepsilon\,, \tag{2.10}$$

$$\delta\lambda = \left(-\frac{1}{2a}\gamma^\mu P_\mu - 2\epsilon m a\, CS\right)\varepsilon\,. \tag{2.11}$$

With these preliminaries, we now search for supersymmetric solutions of this model in the next section.

3. Supersymmetric string and wave solutions

Our metric ansatz is

$$ds^2 = -F^2dt^2 + H^2(Gdt+d\theta)^2 + dr^2\,, \tag{3.1}$$

where F, G and H are functions of r only. We choose the dreibeins of this metric as

$$e_{r1} = 1\,,\qquad e_{t0} = F\,,\qquad e_{t2} = GH\,,\qquad e_{\theta 2} = H\,,$$

$$e^r{}_1 = 1\,,\qquad e^t{}_0 = -\frac{1}{F}\,,\qquad e^\theta{}_0 = \frac{G}{F}\,,\qquad e^\theta{}_2 = \frac{1}{H}\,, \tag{3.2}$$

and the connection 1-forms turn out to be

$$\omega_t{}^{01} = -F' + \frac{H^2}{2F} G G' \,, \qquad \omega_t{}^{12} = -GH' - \frac{G'H}{2} \,,$$
$$\omega_r{}^{02} = \frac{H}{2F} G' \,, \qquad \omega_\theta{}^{01} = \frac{H^2}{2F} G' \,, \qquad \omega_\theta{}^{12} = -H' \,, \tag{3.3}$$

where prime indicates derivative with respect to r. The determinant of the metric is $\sqrt{-g} = FH$ and the nontrivial components of its inverse are given as

$$g^{tt} = -\frac{1}{F^2} \,, \qquad g^{\theta t} = \frac{G}{F^2} \,, \qquad g^{rr} = 1 \,, \qquad g^{\theta\theta} = \frac{1}{H^2} - \frac{G^2}{F^2} \,. \tag{3.4}$$

We choose the scalar field to be of the form

$$\phi = R(r) e^{in\theta} e^{ikt} \,, \tag{3.5}$$

where n and k are real constants. For the vector field we pick the following gauge

$$A_\mu = (A_t, A_r, A_\theta) = (\psi(r), 0, \chi(r)) \,. \tag{3.6}$$

All fermions are set to zero. However in order to obtain a supersymmetric solution we still need to solve (2.10) and (2.11). Solutions can be divided into two classes: $a^2 \neq 0$ and $a^2 = 0$.

3.1 Non-linear sigma models ($a^2 \neq 0$)

From $\delta\lambda = 0$, we find

$$\left[\frac{i(k - \epsilon\psi)S}{F} \gamma_0 - \left(\frac{G}{F}\gamma_0 - \frac{1}{H}\gamma_2 \right) i(n - \epsilon\chi)S + \frac{R'}{R} S\gamma_1 + 4\epsilon m a^2 C S \right] \varepsilon = 0 \,, \tag{3.7}$$

and from $\delta\psi_\mu = 0$, we get

$$\partial_t \varepsilon = \left[\frac{iQ_t}{2a^2} - \left(\frac{F'}{2} - \frac{H^2 G G'}{4F} + mC^2 GH \right) \gamma_2 + \left(\frac{G'H}{4} + \frac{GH'}{2} + mC^2 F \right) \gamma_0 \right] \varepsilon \tag{3.8}$$
$$\partial_\theta \varepsilon = \left[\frac{iQ_\theta}{2a^2} + \left(\frac{H^2 G'}{4F} - mC^2 H \right) \gamma_2 + \frac{H'}{2}\gamma_0 \right] \varepsilon \,, \tag{3.9}$$
$$\partial_r \varepsilon = -\left[\frac{HG'}{4F} + mC^2 \right] \gamma_1 \varepsilon \,. \tag{3.10}$$

For a 1/2 supersymmetric solution we assume a projection of the form

$$\gamma_1 \varepsilon = p\, \varepsilon \,, \qquad (p^2 = 1) \,. \tag{3.11}$$

The above projection applied to (3.7) gives (note that $\gamma_0 = -\gamma_2\gamma_1$)

$$\frac{R'}{R} = -4p\epsilon m a^2 C \,, \tag{3.12}$$
$$\frac{\epsilon}{F}(\psi - G\chi) = -\frac{1}{F}(Gn - k) - \frac{p}{H}(n - \epsilon\chi) \,. \tag{3.13}$$

Equation (3.12) is integrable and the result is

$$C^2 = \frac{1}{1 + 4\epsilon R_0^2 e^{-8\epsilon p m a^2 r}}\,, \tag{3.14}$$

where $2R_0$ is an integration constant and can be set to 1 by a shift in r. The r dependence of the functions R and S can be obtained using (2.5). When $\epsilon = 1$ the range of the coordinate r is $(-\infty, \infty)$, and when $\epsilon = -1$ it is $[0, -p\infty)$. The scalar field is smooth when the r-coordinate is in these intervals. Now using the projection (3.11) in (3.8) and (3.9), we find that the metric functions should obey

$$\frac{H'}{H} = \frac{pG'H}{2F} - 2pmC^2\,, \tag{3.15}$$

$$\frac{F'}{F} = -\frac{pG'H}{2F} - 2pmC^2\,. \tag{3.16}$$

In order to determine the Killing spinor, we begin from (3.10) which fixes its form as

$$\varepsilon = \sqrt{F} e^{\sigma(t,\theta)}\,, \tag{3.17}$$

where we used (3.16) too. Inserting this result in (3.9) we get

$$\frac{\partial\sigma}{\partial\theta} = \frac{i}{2a^2}\left(\frac{2nR^2}{1+\epsilon R^2} + \chi C\right)\,. \tag{3.18}$$

Since the right-hand side of this equation is a function of r only we conclude that

$$\frac{\partial\sigma}{\partial\theta} = ic_1\,, \tag{3.19}$$

where c_1 is a real constant. A similar analysis of (3.8) gives

$$\frac{\partial\sigma}{\partial t} = \frac{i}{2a^2}\left(\frac{2kR^2}{1+\epsilon R^2} + \psi C\right)\,, \tag{3.20}$$

which implies

$$\frac{\partial\sigma}{\partial t} = ic_2\,, \tag{3.21}$$

where c_2 is another constant. The final expressions for the Killing spinor and vector field components are

$$\varepsilon = \sqrt{F} e^{ic_1\theta} e^{ic_2 t}(p + \gamma_1)\varepsilon_0\,, \tag{3.22}$$

$$\chi = \epsilon n - \frac{\epsilon(n - 2\epsilon c_1 a^2)}{C}\,, \tag{3.23}$$

$$\psi = \epsilon k - \frac{\epsilon(k - 2\epsilon c_2 a^2)}{C}\,, \tag{3.24}$$

where ε_0 is a constant spinor. Notice that when the spinor and the scalar field (3.5) are real, the vector field vanishes. This is the chargeless limit of our solutions. Equations (3.23) and (3.24) together with (3.13) put a strong restriction on the metric functions

$$k - 2c_2 a^2\epsilon = (n - 2c_1 a^2\epsilon)\left(G + \frac{pF}{H}\right)\,. \tag{3.25}$$

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This completes our investigation of the supersymmetry variations. Now we have to check the field equations. The scalar field equation (2.9) is identically satisfied. The vector field equation (2.8) is trivially satisfied when the free index $\mu = r$ due to the fact that $\mathrm{Im}\{P^r S^*\} = 0$. However when $\mu = t$ and $\mu = \theta$, we get

$$\chi' = 4\epsilon pma^2|S|^2(n - \epsilon\chi)\,, \tag{3.26}$$

$$\psi' = 4\epsilon pma^2|S|^2(k - \epsilon\psi)\,, \tag{3.27}$$

where we also used (3.13). These two equations have to be consistent with the supersymmetric forms of χ and ψ given in (3.23) and (3.24) which is indeed the case. Finally, after a lengthy calculation Einstein's equations (2.7) can be shown to be satisfied provided that the following is true:

$$p\left(\frac{HG'}{F}\right)' + \left(\frac{HG'}{F}\right)^2 = 4mC^2\frac{HG'}{F} - \frac{2|S|^2}{a^2}\left(\frac{n-\epsilon\chi}{H}\right)^2. \tag{3.28}$$

To summarize, 1/2 supersymmetry breaking projection (3.11) completely determines the scalar and vector fields. It only remains to solve equations (3.15), (3.16) and (3.28) with the condition (3.25) which we do next. Equations (3.15) and (3.16) can be used to determine the metric functions G and F in terms of H as:

$$F = \frac{f_0|S|^{\epsilon/a^2}}{H}\,, \qquad G = -\frac{pF}{H} + g_0\,, \tag{3.29}$$

where f_0 and g_0 are real integration constants. Note that f_0 can never be zero. In obtaining this, we used definitions of C and S (2.4) together with (3.12) to write

$$C' = 4pma^2C|S|^2\,, \qquad |S|' = -4\epsilon pma^2C^2|S|\,. \tag{3.30}$$

The restriction (3.25) fixes the constant g_0 in (3.29) as

$$k - 2c_2a^2\epsilon = (n - 2c_1a^2\epsilon)g_0\,. \tag{3.31}$$

When $(g_0 \neq 0)$ this relation induces an electromagnetic self-duality condition

$$\psi - \epsilon k = g_0(\chi - \epsilon n) \qquad \text{or} \qquad E = -g_0B\,, \tag{3.32}$$

where we denoted the components of the electromagnetic field tensor in the orthonormal basis as $F_{01} = E$ and $F_{12} = B$. Now from (3.15) and (3.16), we have

$$\frac{HG'}{F} = p\left(\frac{2H'}{H} + 4pmC^2\right). \tag{3.33}$$

Inserting this and (3.30) in (3.28), we finally obtain

$$(H^2)' + 4pmC^2H^2 = \frac{(n - 2\epsilon c_1a^2)^2}{4pma^4C^2} + c_0\,, \tag{3.34}$$

where c_0 is a real constant. Now our problem is reduced to a single linear, ordinary, first order differential equation for H^2. The most general solution of (3.34) is

$$H^2 = h_0|S|^{\epsilon/a^2} + h_1 F\left(-\frac{\epsilon}{2a^2}, 1; 1 - \frac{\epsilon}{2a^2}; \epsilon|S|^2\right) + \frac{h_2}{C^2}, \tag{3.35}$$

where h_0 and $h_1 = [c_0/4pm - h_2(1 + 2\epsilon a^2)]$ are arbitrary real constants and

$$h_2 = -\frac{\epsilon(n - 2\epsilon c_1 a^2)^2}{32 m^2 a^6}. \tag{3.36}$$

Here $F(a, b; c; z)$ is a hypergeometric function.[2] Using (3.29) we can write our metric (3.1) as

$$ds^2 = -2pf_0|S|^{\epsilon/a^2} dv dt + H^2 dv^2 + dr^2, \qquad v \equiv \theta + g_0 t. \tag{3.37}$$

It is clear that constants p and f_0 can be discarded by redefining the t coordinate in (3.37). Furthermore, the magnitude of one of the integration constants in H^2 can be set to 1 by a rescaling of the coordinates v and t.

Let us now comment on each term in H^2. When $h_1 = h_2 = 0$ using (3.35) in (3.29) we see that G is constant. (Note that $h_2 = 0$ implies that the vector field is constant too, i.e. $\chi = \epsilon n = 2c_1 a^2$ and $\psi = \epsilon k = 2c_2 a^2$.) In this case the metric (3.37) becomes

$$ds^2 = h_0|S|^{\epsilon/a^2}\left[-\frac{f_0^2}{h_0^2}dt^2 + \left(d\theta + \left[g_0 - \frac{pf_0}{h_0}\right]dt\right)^2\right] + dr^2. \tag{3.38}$$

When $G = g_0 - pf_0/h_0 = 0$ this solution represents a static string. The uncharged version of this solution, i.e. $\chi = \psi = 0$ or $n = k = c_1 = c_2 = 0$, was studied in [15]. When G is equal to a non-zero constant this solution can be interpreted as a rotating (stationary) string provided that the coordinate θ is periodic with $0 \leq \theta < 2\pi$. Actually this metric can be obtained from the static one by a simple boost in the $\theta - t$ plane. However, this transformation is well-defined only locally, and therefore one ends up with a globally stationary solution [33]. Despite the fact that the vector field is constant for this string solution, there is still a non-zero charge since A_μ is a Chern-Simons gauge field. Unlike the Maxwell theory, the charge Q associated with it is obtained by integrating A_μ and not its derivative (see e.g. [30] for a nice discussion of this). Making a coordinate redefinition $(\theta \to \theta - tk/n)$, the time dependence of the scalar field (3.5) and the Killing spinor (3.22) can be turned off [30]. This also sets $A_t = \psi = 0$. Then the charge associated with $A_\theta = \chi$ is

$$Q = 2\epsilon n = 4c_1 a^2. \tag{3.39}$$

Let us emphasize that even when one starts with $h_0 = 0$ in (3.35), by making a coordinate redefinition $(\tau = t + h_0 v/2pf_0)$ this term can still be reintroduced in (3.37). Therefore, the string is always present.

The h_1 term corresponds to nothing but a wave along the string. There is a solution generating technique developed by Garfinkle and Vachaspati [17, 18] which allows one to

[2] The function $F(a, b; c; z)$ satisfies $z(1-z)F'' + [c - (a + b + 1)z]F' - abF = 0$. It has the property $F(1, 1; 2; z) = -z^{-1}\ln(1 - z)$ and $F(\frac{1}{2}, 1; \frac{3}{2}; -z^2) = z^{-1}\arctan z$.

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add such a wave to an already constructed solution when there is a null Killing vector. This process does not affect the matter fields. (For a short outline of this method and its application to our case, we refer the reader to the appendix.)

Finally, the h_2 part is related to an electromagnetic wave. Note that $h_2 \neq 0$ only when the vector field has a radial dependence as can be seen from (3.23), (3.24), (3.31) and (3.36). One may think that when $h_0 = h_1 = 0$, a solution with $h_2 \neq 0$ exists only for $\epsilon = -1$ since then H^2 becomes negative, but in fact this is not necessary. In this case it can be seen from (3.1) and (3.29) that t is no longer the timelike coordinate yet the metric is still well-defined.

Let us now analyze the singularity structure of our solution. There are three curvature invariants in $D = 3$:

$$\begin{aligned} g^{\mu\nu} R_{\mu\nu} &= -8m^2 C^2 (3C^2 - 8a^2 |S|^2)\,, \\ R^{\mu\nu} R_{\mu\nu} &= 64 m^4 C^4 (3C^4 - 16 a^2 C^2 |S|^2 + 24 a^4 |S|^4)\,, \\ \frac{\det(R_{\mu\nu})}{\sqrt{-g}} &= 512 m^6 C^6 (C^2 - 2a^2|S|^2)^2 (C^2 - 4a^2 |S|^2)\,. \end{aligned} \tag{3.40}$$

One should also remember that in $D = 3$, the Riemann tensor is completely determined in terms of the Ricci tensor. Now we proceed with our investigation of $\epsilon = -1$ and $\epsilon = 1$ cases separately.

3.1.1 Non-compact sigma manifold ($\epsilon = -1$)

It is easy to see from (3.40) that a curvature singularity appears as $C^2 \to \infty$ which implies $|\phi| \to 1$ from (2.5). To find out whether there is any horizon, we define a new radial coordinate

$$\rho = \frac{1}{C^2 - 1}\,, \qquad 0 \leq \rho < \infty\,. \tag{3.41}$$

Now the metric (3.37) becomes

$$ds^2 = -2p f_0 \rho^{\frac{1}{2a^2}} dv dt + H^2 dv^2 + \frac{d\rho^2}{64 m^2 a^4 (\rho + 1)^2}\,, \tag{3.42}$$

where

$$H^2 = h_0 \rho^{\frac{1}{2a^2}} + h_1 F\left(\frac{1}{2a^2}, 1; 1 + \frac{1}{2a^2}; -\frac{1}{\rho}\right) + h_2 \frac{\rho}{1+\rho}\,. \tag{3.43}$$

An inspection of the zeros of $g^{\rho\rho} = -f_0^2 \rho^{1/a^2}$ shows that there is no horizon and we have a naked singularity at $\rho = 0$ (or $C^2 \to \infty$).

From the curvature scalar (3.40) and the Ricci tensor (2.7), it is observed that when $C^2 \to 1$, i.e. $|\phi| \to 0$, the solution becomes locally AdS_3. In fact when the waves are present the metric corresponds to a generalized Kaigorodov metric [19] as was studied in [21]; otherwise the asymptotic geometry is AdS_3 [15]. From (3.23) and (3.24), one can see that the vector field becomes constant both at the singularity and at the asymptotic region.

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3.1.2 Compact sigma manifold ($\epsilon = 1$)

Let us now define a new radial coordinate

$$\rho = \frac{M}{C^2}\,, \qquad M \leq \rho < \infty\,, \tag{3.44}$$

where M is a positive constant. Now the metric (3.37) becomes

$$ds^2 = -2pf_0\left(1-\frac{M}{\rho}\right)^{\frac{1}{2a^2}} dvdt + H^2dv^2 + \frac{d\rho^2}{64m^2a^4(\rho-M)^2}\,, \tag{3.45}$$

where

$$H^2 = h_0\left(1-\frac{M}{\rho}\right)^{\frac{1}{2a^2}} + h_1F\left(-\frac{1}{2a^2},1;1-\frac{1}{2a^2};1-\frac{M}{\rho}\right) + h_2\frac{\rho}{M}\,. \tag{3.46}$$

We see that there is a horizon at $\rho = M$ (or $C = 1$). As $\rho \to M$, the curvature scalar (3.40) becomes constant and the geometry is observed to be locally AdS which is a Kaigorodov [19] type of space if the waves are present. When there is only the string, the near horizon geometry is AdS_3 [15]. In this limit $H^2 \to h_1 + h_2$ and the vector field (3.23), (3.24) also becomes constant.

As above there is a curvature singularity as $C^2 \to \infty$, i.e. $\rho \to 0$, but we know from (2.5) that $C \leq 1$ and therefore the singularity is not accessible. Additionally, when $h_2 = 0$, for $C^2 \to 0$ (or $\rho \to \infty$) the solution is asymptotically flat. However, even when $h_2 \neq 0$ in the asymptotic regime, all curvature invariants (3.40) still vanish and the metric becomes

$$ds^2 \to -2pf_0dvdt + \frac{h_2}{M}\rho dv^2 + \frac{d\rho^2}{64m^2a^4\rho^2}\,, \tag{3.47}$$

which describes a pp-wave geometry. Therefore the solution with h_2 term present might be interpreted as a string in a space filled with electromagnetic radiation in the asymptotic regime. Note that in this limit the vector field (3.23), (3.24) diverges.

Now let us look at the behavior of the geodesics. The geodesic equation associated with the metric (3.45) is:

$$\frac{1}{64m^2a^4}\left(\frac{\dot\rho}{\rho}\right)^2 = \alpha\left(1-\frac{M}{\rho}\right)^2 - 2EP\left(1-\frac{M}{\rho}\right)^{2-\frac{1}{2a^2}} + E^2H^2\left(1-\frac{M}{\rho}\right)^{2-\frac{1}{a^2}}\,, \tag{3.48}$$

where the dot denotes derivative with respect to an affine parameter and $\alpha = 0$ or $\alpha = -1$ for null or timelike geodesics, respectively. In this equation E and P are the conserved quantities associated with the flow of the tangent vector of a geodesic corresponding to t and v variables.

When $h_2 \neq 0$, the right-hand side of (3.48) becomes negative as $\rho \to \infty$ since $h_2 < 0$ by (3.36). This means that neither timelike nor null geodesics can reach the asymptotic region. Now lets assume that $h_2 = 0$. The h_1 term is well-defined only when $1/a^2$ is an odd integer (remember that $1/a^2$ is an integer when $\epsilon = 1$) since the hypergeometric function diverges otherwise. In the asymptotic and the near horizon limits, this function behaves as

$$\begin{aligned}
&\lim_{\rho\to\infty} F\left(-\frac{1}{2a^2},1;1-\frac{1}{2a^2};1-\frac{M}{\rho}\right) \approx -\frac{1}{2a^2}\log\rho\,,\\
&\lim_{\rho\to M} F\left(-\frac{1}{2a^2},1;1-\frac{1}{2a^2};1-\frac{M}{\rho}\right) \approx 1\,,
\end{aligned} \tag{3.49}$$

and in both limits the last term in the right hand side of (3.48) is the dominant term if $1/a^2 > 1$. If the geodesics are required to be able to reach the $\rho \to \infty$ limit, then h_1 should be negative. However, since the hypergeometric function changes sign as we approach to the horizon ($\rho \to M$), there should be a turning point and the geodesics never reach the horizon. If $a^2 = 1$, the geodesics may reach the horizon for large enough $(E^2 h_1 - 2EP) > 0$, but even in this case they can not go beyond the horizon since the h_1 term does not change sign whereas others do. When $h_1 = h_2 = 0$, the timelike geodesics can not reach the horizon. Moreover, the null geodesics do not cross the horizon unless $1/a^2$ is a multiple of 4. Since the scalar field can not be extended beyond the horizon, there exist physically well-defined strings only for ($1/a^2 = 1, 2, 3 \mod 4$) as was shown in [15]. Before we close this subsection let us note that when $a^2 = 1/2$, the metric of the $h_1 = h_2 = 0$ solution is precisely the string solution obtained in [34] using the low energy limit of a three dimensional string theory [15].

3.2 Flat sigma model ($a^2 = 0$)

To take the $a^2 = 0$ limit in our model, first one has to rescale $A_\mu \to a^2 A_\mu$ and $\phi \to a\phi$. Then we have $C \to 1$, $S \to 2a\phi$, and one obtains $N = 2$, AdS_3 supergravity with cosmological constant $-2m^2$ coupled to an R^2 sigma manifold [15]. This coincides with the flat sigma model limit of the $N = 2$ theory discussed in [30] as was shown in [28]. The lagrangian (2.2) now becomes

$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{4}R - \frac{1}{16m}\frac{\epsilon^{\mu\nu\rho}}{\sqrt{-g}}A_\mu \partial_\nu A_\rho - |\partial_\mu \phi|^2 + 2m^2\right), \tag{3.50}$$

and its fermionic supersymmetry transformations are

$$\delta\psi_\mu = \left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab} - \frac{i}{2}\Big[i\phi \overleftrightarrow{\partial_\mu} \phi^* + A_\mu\Big]\right)\varepsilon + m\gamma_\mu \varepsilon\,, \tag{3.51}$$

$$\delta\lambda = -(\gamma^\mu \partial_\mu \phi)\varepsilon\,. \tag{3.52}$$

To find a 1/2 supersymmetric solution, we again choose the same metric ansatz (3.1) and use the same form of scalar and vector fields given in (3.5) and (3.6). Now using the projection condition (3.11) in $\delta\lambda = 0$, we find

$$R' = 0\,, \qquad G = -\frac{pF}{H} + \frac{k}{n} \tag{3.53}$$

from which we write

$$\phi = R_0 e^{in\theta} e^{ikt}\,, \tag{3.54}$$

where R_0 is a constant. Equations (3.15) and (3.16) obtained from $\delta\psi = 0$ are still valid with $C = 1$, which gives

$$F = \frac{f_0 e^{-4pmr}}{H}\,, \tag{3.55}$$

where f_0 is a non-zero integration constant. Now our metric can be written as

$$ds^2 = -2pf_0 e^{-4pmr} dv dt + H^2 dv^2 + dr^2\,, \qquad v \equiv \theta + \frac{k}{n}t\,. \tag{3.56}$$

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From $\delta\psi = 0$, we also identify the Killing spinor and vector field components as:

$$\varepsilon = \sqrt{F} e^{ic_1\theta} e^{ic_2 t} (p + \gamma_1)\varepsilon_0 \,, \tag{3.57}$$

$$\chi = 2c_1 - 2nR_0^2 \,, \tag{3.58}$$

$$\psi = 2c_2 - 2kR_0^2 \,, \tag{3.59}$$

where c_1 and c_2 are real constants. The vector (2.8) and the scalar (2.9) field equations are automatically satisfied, but Einstein's equations (2.7) impose:

$$(H^2)' + 4pmH^2 = -8R_0^2 n^2 r + c_0 \,, \tag{3.60}$$

where c_0 is a constant. The most general solution of this equation is

$$H^2 = h_0 e^{-4pmr} + h_1 + h_2 r \,, \tag{3.61}$$

where h_0 and $h_1 = (pmc_0 + 2R_0^2 n^2)/4m^2$ are arbitrary real constants and we have

$$h_2 = -\frac{2R_0^2 n^2 p}{m} \,. \tag{3.62}$$

From the curvature invariants (3.40), we see that the solution has constant negative curvature and it is locally AdS_3. Furthermore there is no curvature singularity. When $h_1 = h_2 = 0$, the metric is the AdS_3 metric in Poincaré coordinates. For this case even when $h_0 = 0$, one can still identify AdS_3 by a simple coordinate redefinition. The h_1 term can be obtained by using the Garfinkle-Vachaspati method [17, 18] (see the appendix) and it describes a wave in AdS_3. Actually the metric with $h_2 = 0$ has already been discussed in [21] and it corresponds to a generalized Kaigorodov metric [19]. It is obtainable from the AdS_3 metric by an $SL(2, R)$ transformation [22] and its equivalance to the extreme BTZ black hole [20] can be shown [21, 22]. When $h_2 \neq 0$, the constant h_1 can be removed by a shift in r. This spacetime is another pp-wave in AdS_3. From (3.62) it is clear that it exists only for a non-zero scalar field. Finally, we would like to point out that our solutions with waves preserve 1/2 supersymmetry similar to those found in [27, 26] for $D = 4, 5$ AdS gauged supergravities.

4. Conclusions

In this section we would like to make some further remarks about our results and suggest some open problems. As we have noted before, the waves alter the global structure and in several instances the Kaigorodov space [19] or some other pp-wave background emerge. This makes our solutions suitable for understanding the AdS/CFT duality in the infinite momentum frame [21, 22] and in the BMN type limit [5]. For example there is still no rigorous notion of ADM mass or energy for such spacetimes and our solutions might be useful in this respect. Another interesting project is to repeat the RG flow analysis of [29] for the charged strings obtained in this paper.

The presence of a horizon for the $\epsilon = 1$ case is very suggestive. Although the curvature invariants turn out to be finite at the horizon one has to be cautious about singularities.

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As was shown in [35, 36, 37, 22] there may still be infinities such as the tidal forces felt by an infalling observer. It would be nice to make a more detailed analysis of this and to see whether the no hair conjecture is supported or not. For this purpose it may be necessary to generalize our solutions to have non-constant wave profiles. It is not clear whether they would still be supersymmetric.

The waves we found are anchored to the string and when $\epsilon = 1$ and $h_2 = 0$ the solution is asymptotically flat. This makes it useful for trying to explain the Bekenstein-Hawking entropy by counting the BPS microstates [38] as was illustrated in [39] for a similar set-up. Of course to do this, first higher dimensional origin of our model has to be identified.

The electromagnetic waves that we obtained do not affect the behaviour of the scalar field. This perhaps hints to a solution generating mechanism within our model like the Garfinkle-Vachaspati method. Another example of such a technique can be found in [13] where solutions were constructed by applying a sequence of manipulations called the null Melvin twist.

Another attractive avenue is to look for new supersymmetric solutions of the model. Especially finding black holes would be very appealing. The flat sigma model limit ($a^2 \to 0$) of our theory is related to the model discussed in [30] where some black hole solutions were found. It is therefore quite likely that such a solution exists within our model and it would be very interesting to see the effect of non-linear sigma manifolds on the solution presented in [30]. We hope to report on this issue soon.

Acknowledgments

We would like to thank Gary Gibbons, Rahmi Güven, Ali Kaya, Chris Pope and Ergin Sezgin for useful discussions. This work is partially supported by the Scientific and Technical Research Council of Turkey.

A. Garfinkle-Vachaspati method

In this appendix we would like to give a brief review of the solution generating technique developed by Garfinkle and Vachaspati [17, 18] and apply it to our string solution (3.38). A detailed discussion of this method together with its extension to various supergravity theories can be found in [35]. Let g_{ab} be a solution of Einstein's equations with some matter source in a given dimension. Let the metric g_{ab} have a null, hypersurface orthogonal Killing vector λ^a. Then one can find a scalar Ω such that

$$\nabla_a \lambda_b = \lambda_{[a} \nabla_{b]} \ln \Omega \,. \tag{A.1}$$

Now a new metric $\hat{g}_{ab}$ defined by

$$\hat{g}_{ab} \equiv g_{ab} + \Omega\, \Phi\, \lambda_a\, \lambda_b \tag{A.2}$$

yields a new solution to the initial theory with the same matter fields as in the background solution. The scalar Φ here satisfies

$$\lambda^a\, \nabla_a \Phi = 0\,, \qquad \nabla^a \nabla_a \Phi = 0\,. \tag{A.3}$$

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This implies λ^a to be a Killing vector also for the new solution $\hat{g}_{ab}$. Hence, the new solution describes a traveling wave since any disturbance must propagate without changing its profile at the speed of light.

In our case we begin from the string metric (3.38) which can easily be put into the form (this is equivalent to starting from (3.37) with $H = 0$)

$$ds^2 = -2pf_0|S|^{\epsilon/a^2} dvdt + dr^2\,, \qquad v \equiv \theta + g_0 t\,, \tag{A.4}$$

where we chose $h_0 = 0$ without any loss of generality. Now using its null Killing vector $k^a = (\partial/\partial t)^a$, the scalar Ω is calculated easily to be $\Omega = \Omega_0 |S|^{-\epsilon/a^2}$, where Ω_0 is an arbitrary constant. Because of (A.3) Φ does not depend on t and is a function of r and v only. Using (3.30), one can explicitly find $\Phi(r,v)$ to be

$$\Phi = \frac{p\Phi_0(v)}{4m}\,|S|^{-\epsilon/a^2}\, F\left(-\frac{\epsilon}{2a^2}, 1; 1 - \frac{\epsilon}{2a^2}; \epsilon|S|^2\right) + \Phi_1(v)\,, \tag{A.5}$$

where Φ_0 and Φ_1 are arbitrary functions of v and they describe the profile of the wave. The difference between the new and the old metric (A.2) is exactly the h_1 term in H^2 (3.35) where the wave profile is fixed to be a constant. This proves our identification of the h_1 term as a string wave. Repeating similar steps for the $a^2 = 0$ case one again finds the h_1 term in (3.61).

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